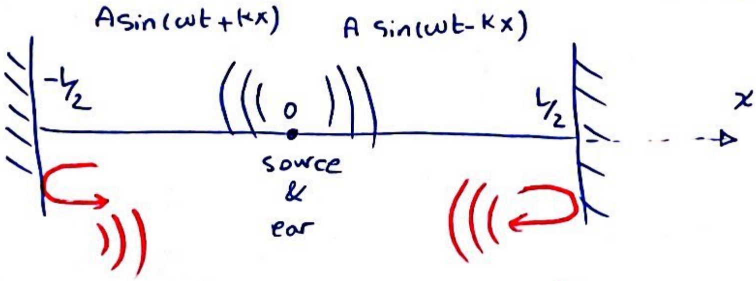
• wave propagetes as $A\sin(\omega t \pm kx)$; $\frac{\omega}{k} = V_{acoustic} \simeq 300\ m/s$



$-A\sin(\omega t + kL - kx)$

$A\sin(\omega t - \frac{kL}{2} + \pi + k(-\frac{L}{2} + x))$

$= -A\sin(\omega t - kL + kx)$

• After the reflections get to $x=0$: [How long it takes: $T = \frac{L}{V_{acoustic}}$]

↳ As if we now have two sources:

↳ @ $x=0$: $A\sin(\omega t) - A\sin(\omega t - kL)$

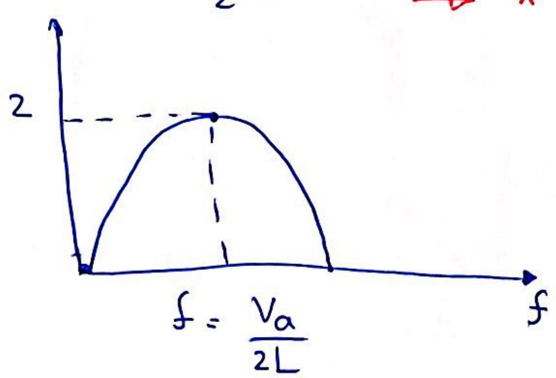
$= 2A\cos(\omega t - \frac{kL}{2})\sin(\frac{kL}{2})$

• Lets plot the amplitude of the new (i.e., equivalent sources):

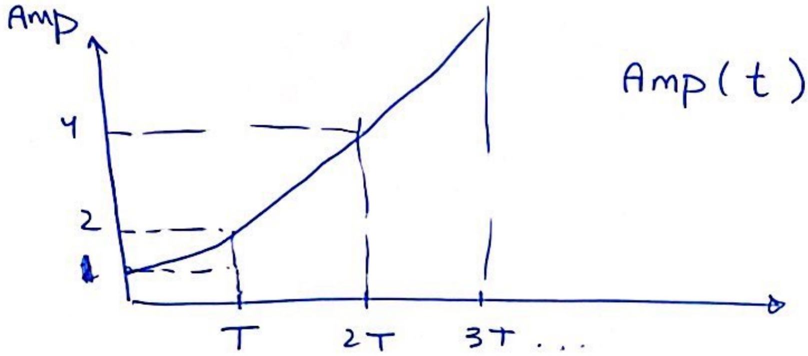
$Amp = 2\sin(\frac{kL}{2})$

↳ * at $\frac{kL}{2} = \frac{\pi}{2}$ Amp becomes double!

$\frac{\omega}{V_a} \cdot L = \pi \rightarrow f = \frac{V_a}{2L}$

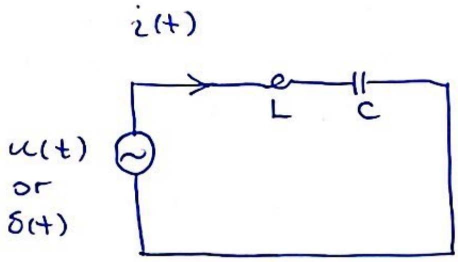


• Take away: if the tone is at $f = \frac{V_a}{2L}$, after each $T = \frac{L}{V_{acoustic}}$ source amp. becomes double! [Re-sound]

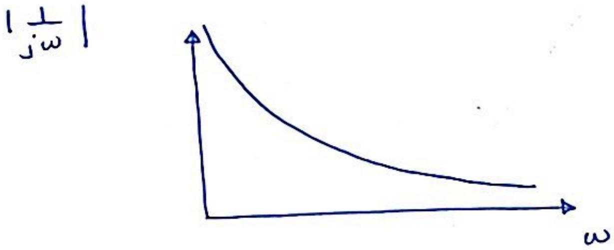


$$Amp(t) = \underbrace{2^{t/T}}_{}$$

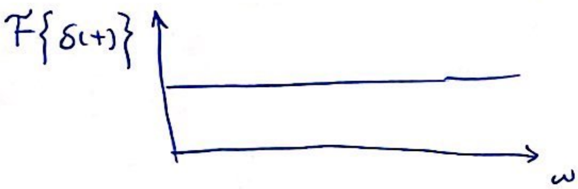
Resonate (Re-Sound!)



- One way to see if a system has resonance (mode) or not is using step or impulse input.
- $u(t)$ & $\delta(t)$, both, has all the freq.



$$\mathcal{F}\{u(t)\} = \frac{1}{j\omega}$$



$$\mathcal{F}\{\delta(t)\} = 1$$

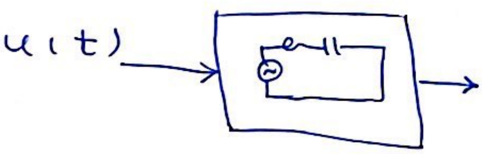
- for $u(t)$ input:

$$\begin{cases} L\dfrac{d^2 i}{dt^2} + \dfrac{1}{C} i = 0 \\ i(0^+) = 0 \\ \dfrac{di}{dt}(0^+) = \dfrac{1}{L} \end{cases} \rightarrow \text{Sol: } i(t) = A\sin(\omega t + \varphi) \rightarrow -L\omega^2 + \frac{1}{C} = 0 \rightarrow \boxed{\omega = \frac{1}{\sqrt{LC}}}$$

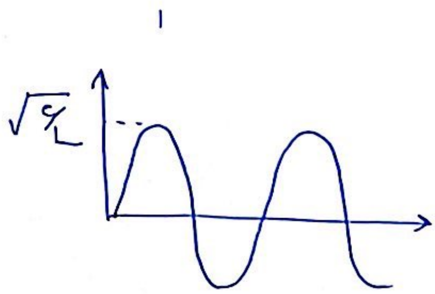
$$i(0) = A\sin(\varphi) = 0 \rightarrow \boxed{\varphi = 0}$$

$$\frac{di}{dt}(0) = A\omega\cos(0) = \frac{1}{L} \rightarrow A = \sqrt{\frac{C}{L}}$$

$$\Rightarrow i(t) = \sqrt{C/L}\,\sin\left(\frac{1}{\sqrt{LC}}\,t\right)$$

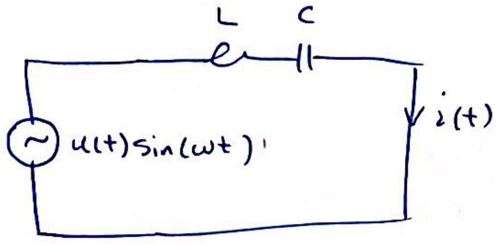


$\rightarrow \sqrt{C/L}\,\sin\left(\frac{1}{\sqrt{LC}}\,t\right)$



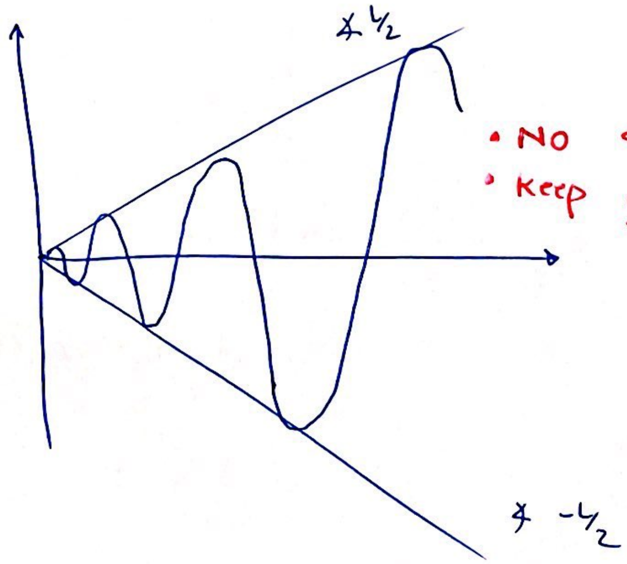
* To find the resonance freq. of a system, apply impulse & look into output @ time domain.

- Excitation at resonance :



$$\begin{cases} L \dfrac{d^2 i}{dt^2} + \dfrac{1}{C} i = \omega \cos(\omega t) \\ i(0) = 0 \\ \dfrac{di}{dt}(0) = 0 \end{cases}$$

$$\longrightarrow \quad i(t) = \frac{L}{2} \, t \, \sin(\omega t)$$



- No steady-state response
- Keep growing !

- Excitation at non-resonance :